A Novel Nonlinear Analysis Tool: Multi-scale Symbolic Sample Entropy and Its Application in Condition Monitoring of Rotary Machinery

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Abstract—**Sample entropy (SE) has been employed for fault diagnosis of rotary machinery (FDRM). However, SE has low computation efficiency for long time series. To solve this problem, symbolic sample entropy (SSE), a novel measure of time series regularity, is proposed to estimate the complexity. However, SSE fails to account for the multiple scale information inherent in measured vibration signals. Therefore, we combine the concept of multi-scale analysis with SSE, called multi-scale SSE (MSSE). To evaluate the effectiveness of the proposed MSSE method, we apply several simulated signals to verify the merits of SSE in impulsion detection and calculation efficiency. Furthermore, we utilize one experimental data to validate its effectiveness in recognizing several fault types of rotary machinery. Experimental results indicate that MSSE has an advantage in extracting fault features compared with multi-scale entropy (MSE), multi-scale fuzzy entropy (MFE), and multi-scale permutation entropy (MPE) methods.**

Keywords- Symbolic sample entropy (SSE); Multi-scale symbolic sample entropy (MSSE); Rotary machinery; Complexity measurement; Feature extraction; Nonlinear.

I. INTRODUCTION

Rotating machinery plays a significant role in mechanical equipment, including many engineering fields such as power, chemical, metallurgy, and machinery manufacturing. It generally works under worse working conditions, which is prone to failures, resulting into machinery sudden shutdown and severe economic loss in industrial application. Therefore, the health condition monitoring (HCM) of rotary machinery is crucial to ensure the safe operation of industrial machinery[1][2][3].

Until now, there are many mature techniques for HCM of rotary machinery. Among these techniques, the vibration-based method is most widely applied in industrial applications[4]. The vibration signal of healthy machinery will have a different entropy value with that of faulty machinery due to the changes of the dynamical complexity. Hence, entropy is an effective method in detecting various faults of rotary machinery. In recent years, various entropies have been developed rapidly to measure the complexity of time series generated from nonlinear dynamical systems. Pincus proposed a family of statistics, called approximate entropy (AE), to measure the regularity of a time series[5]. AE has been widely applied to vibration signal analysis. However, AE is a biased statistic. AE strongly depends on the data length and lacks relative consistency in some cases. To remove the deficiencies, a modification of AE known as sample entropy (SE) was proposed by Richman et al.[6]. SE relieves the bias caused by self-matched so that SE displays relative consistency and less dependence on data length. However, the results of SE for estimating entropy show high sensitivity to the parameter selection and may be invalid in case of small parameter. Chen et al. proposed fuzzy entropy (FE) as the improvement of SE, which uses the fuzzy set theory to count the states of the orbits in the time series[7]. FE can get more precise entropy estimation, which extracts more fault information from the vibration signal of rotary machinery. Permutation entropy (PE) measures the complexity of the time series through the permutation of the orbits to determine the state probability[8].

However, traditional entropy methods have their own disadvantages. SE and FE are not fast enough for some real-time applications, especially for long signals. PE, though computationally efficient, does not consider the influence of the difference between amplitude values for a given time series. In this work, we introduce a new related measure of time series regularity, symbolic sample entropy (SSE) by incorporating the symbolic dynamic filtering (SDF) into SE. The proposed SSE contains the symbolization process, and analysis of symbolic data is often less sensitive to noise. Further, we combine the concept of multi-scale analysis [9] with SSE, namely multi-scale SSE (MSSE).

The remainder of this paper is organized as follows. The proposed SSE algorithm and MSSE algorithm are introduced in Section Ⅱ. In Section Ⅲ, several simulations are described to demonstrate the effectiveness of the proposed SSE algorithm in impulsion detection and calculation efficiency. Section IV provides the experimental variation using one real dataset. A conclusion is provided in Section V.

II. METHOD

In this section, the concepts of the SSE algorithm and MSSE algorithm are detailed explained in follows.

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A. *Symbolic Sample Entropy (SSE)*

Let $X\{x(n), n=1,2,...,N\}$ represent a time series of length *N*. The defined SSE can be obtained following five steps.

Step 1: Discretize the raw time series $X\{x(n), n=1,2,...,N\}$ into a corresponding sequence of ε symbols $s=\{s_1\ s_2\ \cdots\ s_N\}$. Maximum entropy partitioning (MEP) is applied to complete the symbolization in this paper.

Step 2: Construct the embedding vectors based on the symbol sequence with dimension m by using Eq.(1):

$$\mathbf{s}_i^m = \{\, s_i\ s_{i+1}\ \cdots\ s_{i+m-1}\,\},\ 1\le i\le N-m \qquad (1)$$

Step 3: We call $(\mathbf{s}_i^m, \mathbf{s}_j^m)$, $i \neq j$ an m-dimensional matched vector pair if the two symbol vectors are equal. Let n^m represent the total number of m-dimensional matched vector pairs.

Step 4: Repeat steps (1)–(3) for dimension *m*=*m*+1, and n^{m+1} is obtained to represent the total number of (*m*+1) dimensional matched vector pairs.

Step 5: The SSE is defined as the logarithm of the ratio of n^{m+1} to n^m, which can be expressed as $SSE(X,m,\varepsilon) = -\ln\dfrac{n^{m+1}}{n^m}$.

Algorithm 1 Symbolic Sample Entropy

Input: time series $X\{x(n), n=1,2,...,N\}$, embedding dimension m, and the number of symbols ε.

Output: Symbolic Sample Entropy SSE.

```
Discretize the raw time series with ε symbols.
for m=m,m+1 do
    Construct the embedding vectors based on the symbol sequence, and there are ε^m potential state patterns
    Count quantity of every state pattern in the embedding vectors B^(m)(k), 1 ≤ k ≤ ε^m
    the numbers of matched vector pairs n^m = Σ_{k=1}^{ε^m} B^(m)(k) × (B^(m)(k) − 1)
end for
SSE(X,m,ε) = −ln (n^{m+1} / n^m)
```

The pseudo code of the proposed SSE is shown in Algorithm 1. For a better understanding of the calculation process, Fig.1 gives a clear illustration that how to calculate SSE with m=2, and $\varepsilon=2$. First, construct the symbolization and obtain the symbol sequence {0 1 1 1 1 1, … , 0 0 1 1 1 }. Second, the symbol sequence is reconstructed into {0 1}, {1 1}, {1 1}, … , {0 1}, {1 1} for m=2. Third, count quantity of every state pattern in the embedding vectors, show in the histogram. Then, compute n^m=72. Repeat above steps for dimension m=3, and obtain n^{m+1}=34. Last, get the SSE value, namely $SSE = -\ln\dfrac{n^{m+1}}{n^m}$=0.7503 .

B. *Multi-scale Symbolic Sample Entropy(MSSE)*

SSE is a single-analysis approach. However, the fault features of complex rotary machinery are embedded in multiple time-scale domains. Therefore, we proposed the MSSE algorithm to calculate SSE over a range of scales to represent the complexity of a time series. The multi-scale symbolic sample entropy procedure is shown in Algorithm 2.

Algorithm 2 Multi-scale Symbolic Sample Entropy

Input: time series $X\{x(n), n=1,2,...,N\}$, embedding dimension m, the number of symbols ε, and the scale factor τ.

Output: Multi-scale Symbolic Sample Entropy MSSE.

```
for j=1,2,…,τ do
    generate the consecutive coarse-grained time series y_j^(τ) = {y_{j,1}^(τ) y_{j,2}^(τ) … y_{j,p}^(τ)}, 1 ≤ j ≤ τ
    Compute SSE valve of y_j^(τ)
    Augment the data MSSE_{1:j} = {MSSE_{1:j−1}; SSE(m,r,N,j)}
end for
```

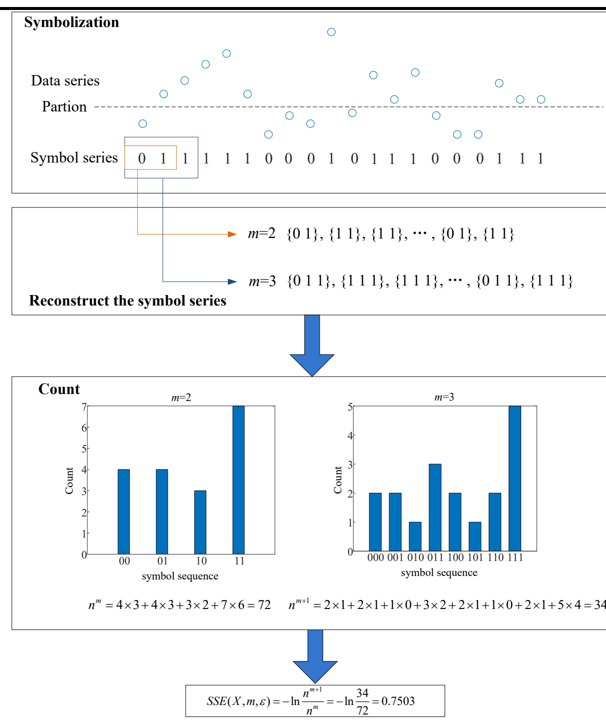

Fig.1 A clear illustration that how to calculate SSE with *m*=2, and $\varepsilon=2$.

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III. Simulation Evaluation

In this section, one impulsive signal is adopted to verify the advantage of proposed SSE in impulsion detection and calculation efficiency.

A. Impulsive detection

To validate the effectiveness of SSE in detecting various fault severities, we use the simulated gear faulty signals with three crack fault severities, including slight fault, medium fault, and severe fault. The synthetic signal has 34816 points, which is cut out by a sliding window of 2048 points with a step length of 512 points. The time domains of the three simulated bearing faulty signals are illustrated shown in Fig.2 (a). For comparison purpose, the SE, FE, and PE are also utilized to process the impulsive signals. Absolute difference (AD) value between the average of the first 5 samples (normal samples) and each of other samples is computed to estimate their fault detection ability. For each sample, here we calculate the SSE with m=2, ε=2, SE with m=2, r=0.15, the PE with m=6, and the FE with m=2, r=0.15.

It can be found that the FE and PE method cannot detect the impulsive derived from slight fault. By contrast, SE and SSE both generate higher AD values when the impulse occurs derived from the slight fault. Moreover, proposed SSE has the least fluctuation for noise, as shown in Fig.2 (e). The phenomenon validates the ability of the proposed SSE in impulsive detection.

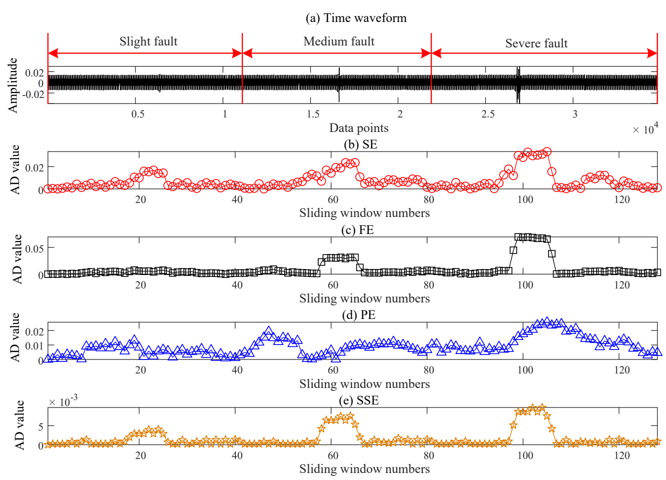

Fig.2 Performance comparison results of SE, FE, PE, and SSE methods: (a) the waveform of the simulated signal (b) the AD value of SE, (b) the AD value of FE, (d) the AD value of PE, (e) the AD value of SSE.

B. Calculation efficiency

To compare the calculation efficiency of SSE, SE, PE, and FE, we calculate the time complexity of the four entropy methods, shown as in TABLE 1. We can know SSE is O(n), PE is O(n), FE is O(n^2), SE is O(n^2).

To intuitively compare the calculation efficiency, we also count the real time consuming of each entropy method. We count the time consuming of each method in Ⅲ.A: SE, 59 seconds; PE, 49.70 seconds; FE, 252.78 seconds; SSE, 2.25 seconds. All the codes are implemented at Matlab R2018a using Core I7-6700HQ @2.6GHz, 16GB RAM. We can find the SE and FE are too time-consuming.

In summary, from the theoretic and simulation results can be seen that SSE is the most time-efficient method.

TABLE I. Calculation Efficiency

Method	SE	PE	FE	SSE
Time complexity	O(n^2)	O(n)	O(n^2)	O(n)
Calculation time(s)	59.00	49.70	252.78	2.25

IV. Experiment Evaluation

In order to verify the effectiveness of the proposed method in actual HCM of rotary machinery, the MSSE is employed to extract the fault features from the vibration signals of rotary machinery. Then, SVM classifier is applied to classify different fault types. The experiments are conducted on rotary machinery called SpectraQuest Machinery Fault Simulator (MFS). The test rig is shown in Fig.3 (a) and Fig.3 (b), respectively. It consists of a reliance electric motor, a three-way gearbox with straight cut bevel gears, and rolling bearings. A magnetic clutch is also mounted at the rear of the gearbox for load generation. An accelerometer is installed on the top of the gearbox to collect the vibration signals. The sampling frequency is set 12800 Hz and the rotating speed is 3000 rpm. In this paper, the load is 5 in-lbs of torque. The different faults are simulated by replaced the fault gear (including pitting in the driving tooth, the broken tooth in the driving tooth and the missing tooth in the driving tooth, as shown in Fig.4 (a), (b), and (c)) and the fault bearing (including inner race fault and outer race fault, shown in Fig.4 (d) and (e)).

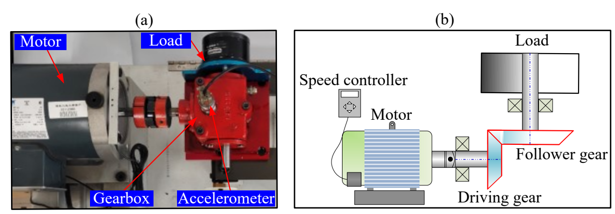

Fig.3 (a) The machinery fault simulator system, (b) the layout of the test rig.

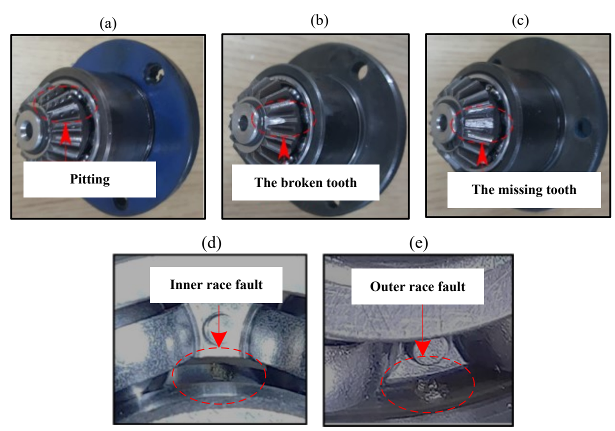

Fig.4 Faulty gears and bearings: (a) pitting in the driving tooth, (b) broken tooth in the driving tooth, (c) missing tooth in the driving tooth, (d) inner race fault, (e) outer race fault

In this section, seven conditions are introduced, which consist of one healthy condition (NOR) and six fault conditions, including pitting in the driving tooth with inner race fault (PI), pitting in the driving tooth with outer race fault (PO), broken tooth in the driving tooth with inner race fault (BI), broken tooth in the driving tooth with outer race fault (BO), missing tooth in the driving tooth with inner race fault (MI), and missing tooth in the driving tooth with outer race fault (MO). Each class owns 100 samples and there are total 700 samples (100 samples × 7 fault types). Meanwhile, the length of each sample is 2048 points. The waveforms under seven healthy conditions are shown in Fig.5. In this section, we set m=2, ε =3 and scale=8 in MSSE.

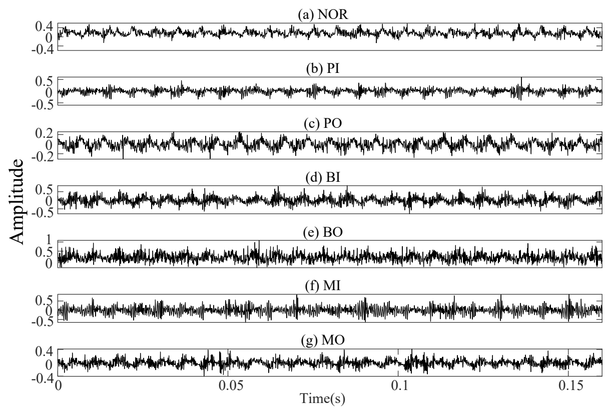

Fig.5 The waveforms under seven working conditions: (a)healthy condition (NOR), (b) pitting in the driving tooth with inner race fault (PI), (c)pitting in the driving tooth with outer race fault (PO), (d)broken tooth in the driving tooth with inner race fault (BI), (e)broken tooth in the driving tooth with outer race fault (BO), (f)missing tooth in the driving tooth with inner race fault (MI), and (g)missing tooth in the driving tooth with outer race fault (MO).

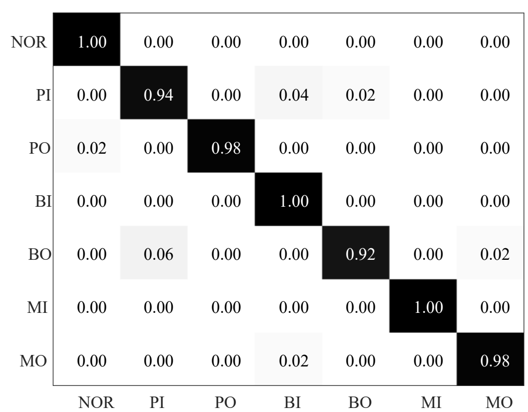

Fig.6 Confusion matrix of recognition using proposed MSSE.

Next, the entropy values of seven health conditions are calculated by MSSE method. In this paper, we randomly choose 50% samples from each health condition for training and residual samples are used to test the classification performance. The classification accuracy is defined as follow:

$$\text{Classification}(\%) = \frac{N_1}{N_2} \times 100 \tag{2}$$

where N_1 represents the correct classified samples and N_2 denotes the total number of samples.

The obtained features are fed into SVM for classification and the obtained results are shown in Fig.6. As can be seen, four working conditions are misclassified. The proposed MSSE method misclassifies 2% testing samples of PO misclassifies as the NOR and 2% testing samples of MO misclassifies as the BI. In addition, 6% testing samples of PI are misclassified as BI and BO. Last, 8% testing samples of BO are misclassified as PI and MO.

For comparison, the MSE, MFE, and MPE are also tested. We set m=2, r=0.15 in MSE, m=6 in MPE, m=2, r=0.15 in MFE and scale=8 for three methods. To avoid randomness, each method runs 20 times and the obtained results are shown in TABLE II.

TABLE II. CLASSIFICATION ACCURACY OF THE EXPERIMENTAL DATA SETS

Method	Accuracy (%)		
	Max	Min	Mean
MSSE	97.14	94.85	96.17
MSE	77.71	68.28	72.34
MFE	95.42	86.85	90.95
MPE	93.14	87.71	90.97

As seen from 0the average testing accuracy of MSSE, MSE, MFE, and MPE is 96.17%, 72.34%, 90.95%, and 90.97%. We can also see that the proposed MSSE has the highest average testing accuracy. This result enforces the conclusion that the proposed MSSE has the best ability to distinguish the health condition of the bearing and gear.

CONCLUSION

In this paper, a new nonlinear dynamic approach called MSSE is proposed to measure the complexity and detect the dynamical changes of time series. Unlike other entropy approaches, MSSE utilizes the SDF to remove the background noises and enhance the computation efficiency. To conclude, there are two main merits in the proposed MSSE. First, robustness. The proposed method is more robust to the noise. Second, high calculation efficiency. The advantages of the proposed MSSE are validated using both the simulated and experiment signals. Results demonstrate that the proposed method can successfully identify different health conditions of the rotating machinery. MSSE can be taken as a promising tool to quantify the complexity of the field data. In further work, we will focus on the parameter selection strategy of MSSE.

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REFERENCES

[1] O. Abdeljaber, O. Avci, S. Kiranyaz, M. Gabbouj, and D. J. Inman, "Real-time vibration-based structural damage detection using one-dimensional convolutional neural networks," *J. Sound Vib.*, vol. 388, pp. 154–170, Feb. 2017, doi: 10.1016/j.jsv.2016.10.043.

[2] Y. Li, X. Wang, Z. Liu, X. Liang, and S. Si, "The Entropy Algorithm and Its Variants in the Fault Diagnosis of Rotating Machinery: A Review," *IEEE Access*, vol. 6, pp. 66723–66741, 2018, doi: 10.1109/ACCESS.2018.2873782.

[3] A. Humeau-Heurtier, "The Multiscale Entropy Algorithm and Its Variants: A Review," *Entropy*, vol. 17, no. 5, pp. 3110–3123, May 2015, doi: 10.3390/e17053110.

[4] P. D. Samuel and D. J. Pines, "A review of vibration-based techniques for helicopter transmission diagnostics," *J. Sound Vib.*, vol. 282, no. 1–2, pp. 475–508, Apr. 2005, doi: 10.1016/j.jsv.2004.02.058.

[5] S. M. Pincus, "Approximate entropy as a measure of system complexity.," *Proc. Natl. Acad. Sci.*, vol. 88, no. 6, pp. 2297–2301, Mar. 1991, doi: 10.1073/pnas.88.6.2297.

[6] J. S. Richman and J. R. Moorman, "Physiological time-series analysis using approximate entropy and sample entropy," *Am. J. Physiol.-Heart Circ. Physiol.*, vol. 278, no. 6, pp. H2039–H2049, Jun. 2000, doi: 10.1152/ajpheart.2000.278.6.H2039.

[7] Weiting Chen, Zhizhong Wang, Hongbo Xie, and Wangxin Yu, "Characterization of Surface EMG Signal Based on Fuzzy Entropy," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 15, no. 2, pp. 266–272, Jun. 2007, doi: 10.1109/TNSRE.2007.897025.

[8] R. Yan, Y. Liu, and R. X. Gao, "Permutation entropy: A nonlinear statistical measure for status characterization of rotary machines," *Mech. Syst. Signal Process.*, vol. 29, pp. 474–484, May 2012, doi: 10.1016/j.ymssp.2011.11.022.

[9] M. Costa, A. L. Goldberger, and C. K. Peng, "Multiscale entropy analysis of complex physiologic time series," *Phys. Rev. Lett.*, vol. 89, no. 6, p. 068102, Aug. 2002, doi: 10.1103/PhysRevLett.89.068102.